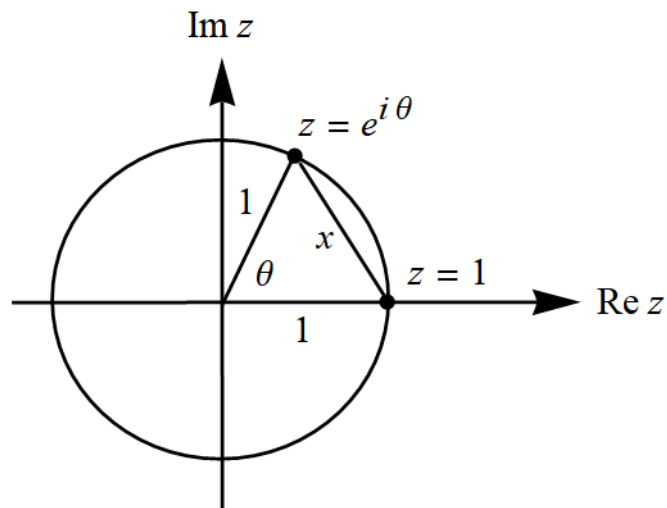


Exercise 4

Using the fact that the modulus $|e^{i\theta} - 1|$ is the distance between the points $e^{i\theta}$ and 1 (see Sec. 4), give a geometric argument to find a value of θ in the interval $0 \leq \theta < 2\pi$ that satisfies the equation $|e^{i\theta} - 1| = 2$.

Ans. π .

Solution

Use the law of cosines.

$$\begin{aligned}x^2 &= 1^2 + 1^2 - 2(1)(1) \cos \theta \\ &= 2 - 2 \cos \theta\end{aligned}$$

Set x , the distance between $z = e^{i\theta}$ and $z = 1$, to 2 and solve for θ .

$$4 = 2 - 2 \cos \theta$$

$$2 = -2 \cos \theta$$

$$\cos \theta = -1$$

$$\theta = \pi + 2\pi n, \quad n = 0, \pm 1, \pm 2, \dots$$

Since we require $0 \leq \theta < 2\pi$, we choose $n = 0$. Therefore,

$$\theta = \pi.$$